Finding structures in the chaos of stratified turbulent flows

STEM for Britain
Mathematical Sciences
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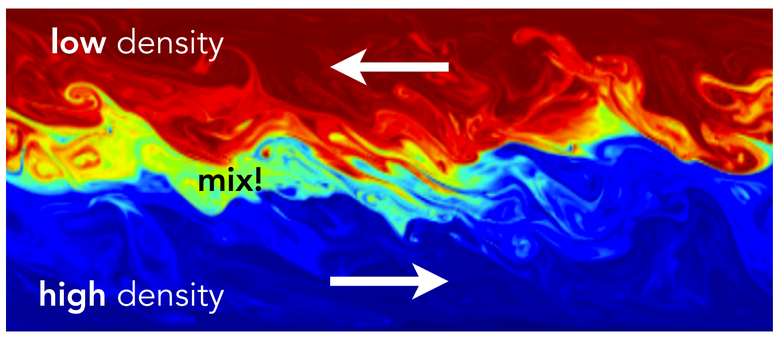
Research topic

Turbulence in density-stratified fluids

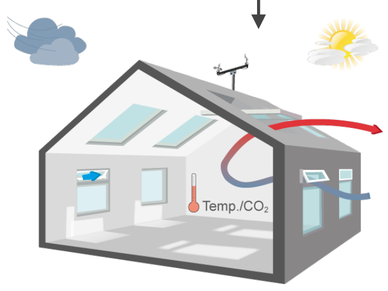


- At high flow speeds, the interface is **turbulent**
- Complex, small-scale, unstable **eddies**
- Transport of salt / heat and momentum = **mixing**
- Mixing **costs energy** ('tax') and **alters the flow**

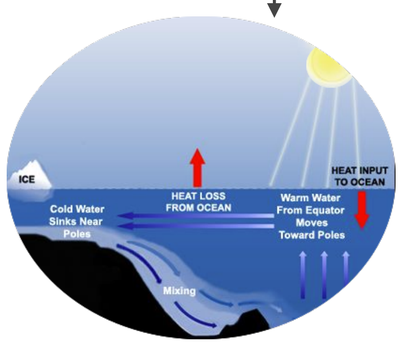
Applications

Predicting the mixing rate

- **Natural ventilation** of buildings



- **Weather** and **climate** simulations



- **Pollutants** dispersion and **air quality**

Challenges

Mathematical modelling

- The **equations** are well-known (Navier-Stokes)

$$\frac{\partial u_i}{\partial t} + u_j \frac{\partial u_i}{\partial x_j} = -\frac{\partial p}{\partial x_i} - Ri\,\rho + \frac{1}{Re}\frac{\partial^2 u_i}{\partial x_j x_j},$$

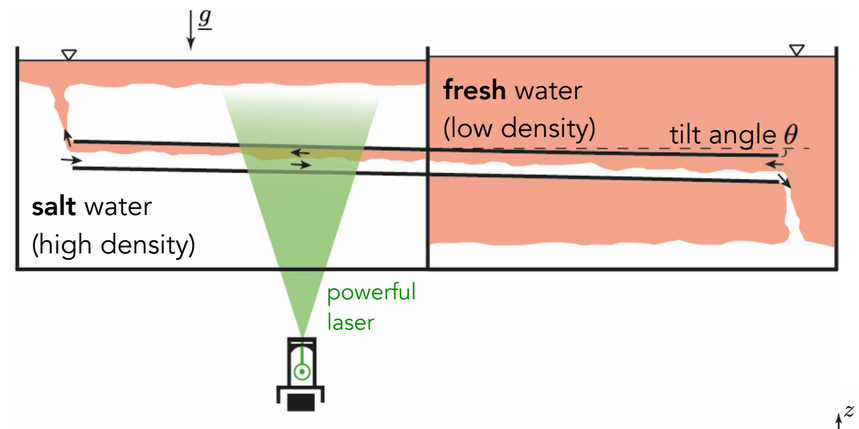
$$\frac{\partial u_j}{\partial x_j} = 0, \qquad \frac{\partial \rho}{\partial t} + u_j \frac{\partial \rho}{\partial x_j} = \frac{1}{Re\ Sc}\frac{\partial^2 \rho}{\partial x_j x_j}$$

x_i, t : 3D spatial coordinates ($i = 1,2,3$) and time
u_i : 3D local velocity components
ρ, p : local pressure and density

- But… coupled nonlinear PDEs with many parameters: **no exact solution** ($1,000,000 prize!)
- Need to **develop simplified models** using **intuition from experiments**

Experimental setup

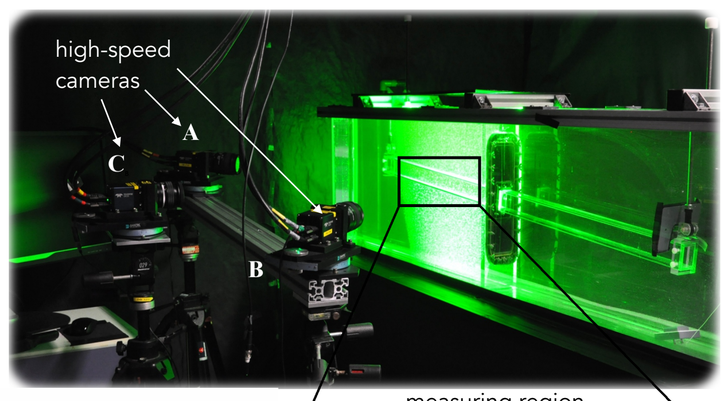
Exchange flow through an inclined duct

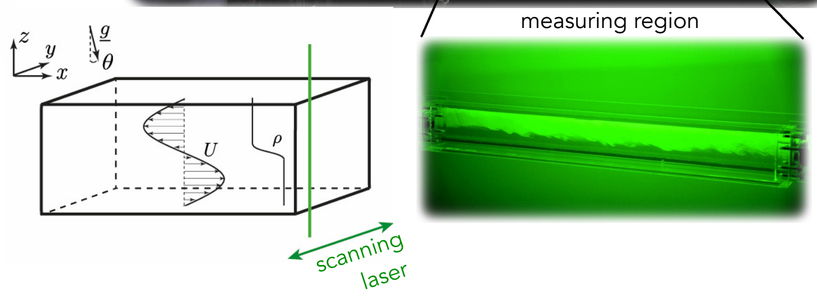


- **Sustains a stratified flow** for long times
- **Excellent model** for many natural flows
- Control over key **flow parameters**
- Extrapolate to full size by **dimensional analysis**

Novel measurements

Velocity and density in a 3D volume!

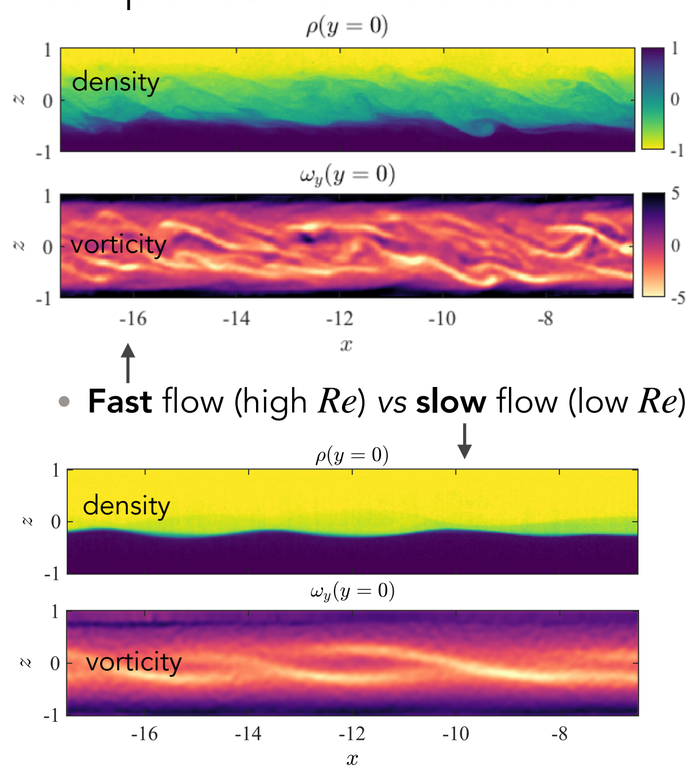




- **Cutting-edge technology** pioneered in our lab

Key observations

Complex 3D flow structures

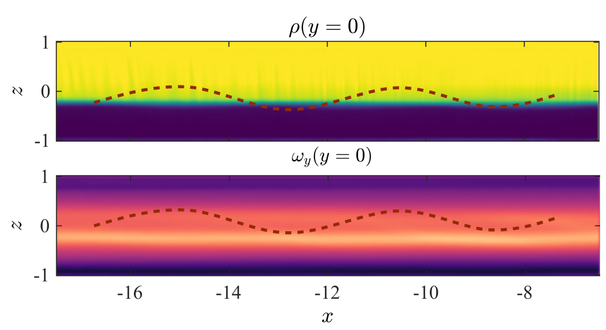


- **Fast** flow (high Re) *vs* **slow** flow (low Re)

Data-driven model

Origin of this slow flow wave structure?

- Does it come from a flow **instability?**
- Combine **3D experimental data**…



… with mathematical **3D linear stability analysis**

$$u_i = \langle u_i \rangle_{x,t}(y,z) + \varepsilon\, \hat{u}_i(y,z)\, \exp(ikx + \sigma t)$$

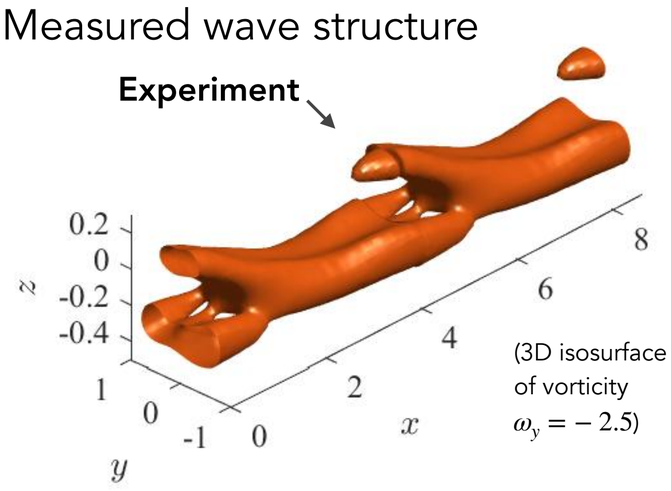
total · base flow · perturbation · structure · growth in time

$$\sigma \begin{bmatrix} \nabla^2 & & \\ & \nabla^2 & \\ & & 1 \end{bmatrix} \begin{bmatrix} \hat{v} \\ \hat{w} \\ \hat{\rho} \end{bmatrix} = \begin{bmatrix} \mathcal{L}_v & \mathcal{L}_{vw} & \mathcal{L}_{v\rho} \\ \mathcal{L}_{wv} & \mathcal{L}_w & \mathcal{L}_{w\rho} \\ & \mathcal{L}_{\rho w} & \mathcal{L}_\rho \end{bmatrix} \begin{bmatrix} \hat{v} \\ \hat{w} \\ \hat{\rho} \end{bmatrix}$$

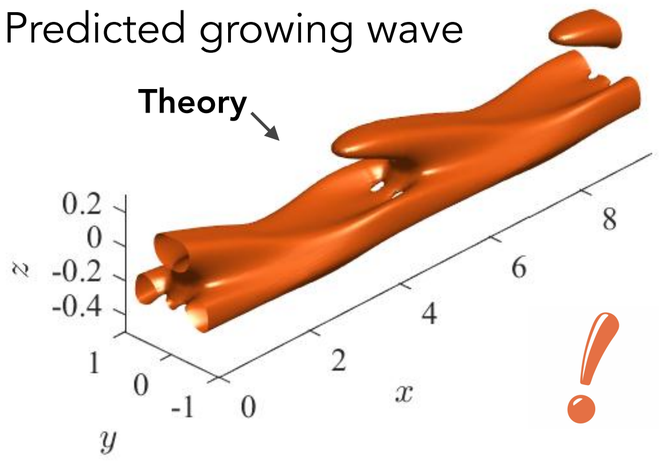
- Solve a **large numerical eigenvalue problem**

Exciting results

Measured wave structure



Predicted growing wave



Originality & Impact

Experiments

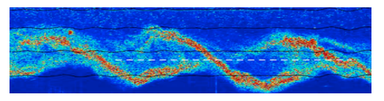
- Unprecedented **measurements** of complex velocity and density data in stratified flows
- Revealed puzzling slow structure: **crucial building block** for fast (turbulent) flows?

Mathematics

- **Data-driven linear stability** on 3D flows
- Explained the **origin and properties** of the slow structure (confined Holmboe mechanism)

Field data

- Possible relevance to **estuarine flows**



Similar structures in Geyer *et al.* (2010)

Find out more

(aim at code with smartphone camera and follow link)

Publication
J. Fluid Mech. ***848***:508-544 *(2018)*

Movie
Outreach movie (3-min long)

Profile

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EPSRC Programme Grant EP/K034529/1
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